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Eighty-two presents were announced as having been received since the last meeting, including, amongst others :—

Agnes Giberne, Sun, Moon, and Stars (new edition), presented by the author ; Lick Observatory publications, No. 3 ; G. Leveau, Théorie du mouvement de Vesta, presented by the author ; J. M. Schaeberle, Terrestrial atmospheric absorption of photographic rays of light, presented by the Observatory ; Original negatives of *Jupiter*, and photographs from drawings of *Mars*, presented by the Lick Observatory.

Reproduction of Photographs.

Arrangements have been made with Messrs. Eyre & Spottiswoode by which Fellows, and the public at large, will be able to obtain reproductions of photographs in the possession of the Society by purchase. Messrs. Eyre & Spottiswoode will copy such photographs, in every case providing one copy for the Society free of cost, and will sell copies to the Fellows or those who apply for them, at a fair price. It is to be understood that this firm has not been granted any exclusive right over the reproduction of photographs, and that the present arrangements are terminable at the pleasure of the Council.

The Council resolved, on 1893 April 14, that notice should be given to the Fellows of these arrangements by a paragraph in the *Monthly Notices*.

E. W. Maunder,
H. H. Turner,
Secretaries.

On the Parallactic Inequality in the Earth's Motion around the Sun. By E. J. Stone, M.A., F.R.S., Radcliffe Observer.

I take for the variations of the geocentric coordinates of the Sun due to a shift of origin from the common centre of gravity of the Earth and Moon to the centre of gravity of the Earth the expressions given by Le Verrier, *Solar Tables*, p. 47 :

$$dr = -\frac{m'}{m+m'} \,.\, r' \cos s' \,.\, \cos(\nu' - \nu).$$

$$\delta s = -\frac{m'}{m+m'}\,\frac{r'}{r} \,.\, \sin s'.$$

$$dr = -\frac{m'}{m+m'}\,\frac{r'}{r} \,.\, \cos s' \,.\, \sin(\nu' - \nu).$$

where r' ν' s' are the geocentric coordinates of the Moon ; r ν the heliocentric coordinates of the Earth ; m' m the masses of the Moon and Sun. Putting $\nu = \odot + 180$, the expressions for $\odot$ and r are taken from Le Verrier's *Solar Tables*, pp. 52, 53, 54.

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The coordinates of the Moon, V, U$=s'$ and P, are taken from Delaunay's numerical results, *Conn. des Temps*, 1869. I put

$$\frac{m'}{m+m'}=\mu;\ \sin 1''\frac{a}{1}=\Pi;\ \sin 1''\frac{a}{r'}=\frac{P}{3422{\cdot}7}\times C,$$

where a is the equatoreal radius of the Earth; π and C are the exact values of the constants of the Solar and Lunar Parallax; and $3422''{\cdot}7$ is the constant of lunar parallax adopted by Delaunay.

I then simply develop the expression

$$\frac{1}{r}\cdot\frac{3422{\cdot}7}{P}\cos s',$$

which I find equals

$$\begin{aligned}0{\cdot}999547\ [1&-0{\cdot}0546\cos l-0{\cdot}0015\cos 2l.+0{\cdot}0020\cos 2F\\&-0{\cdot}0076\cos 2D-0{\cdot}0096\cos(2D-l).\\&-0{\cdot}0004\,.\cos(2D+l)+0{\cdot}0169\,.\cos l'+\&c.]\end{aligned}$$

Denoting $v'-\odot=V-\odot$ by A, and expressing dv in seconds of arc, we have

$$\sin 1''dv''=\frac{\mu\,.\,\Pi}{C}\times 0{\cdot}999547\ [1-0{\cdot}0546\cos l-\&c.]\sin A.$$

If, therefore, we find from a discussion of observations that the coefficient of sin A is an angle a'', we have

$$a''=\frac{\mu\,\Pi}{C\sin 1''}0{\cdot}999547,$$

and therefore, if we adopt for C the value $3422''{\cdot}7$, a value which appears to be one of considerable accuracy, we have

$$\Pi=\frac{a''}{\mu}\,.\,\frac{C\sin 1''}{0{\cdot}999547}=\frac{a''}{\mu}\times 0{\cdot}016601.$$

The factor 0·016601 agrees very closely indeed with Le Verrier's value 0·01662, which I verified and adopted in 1867 to deduce the value of the solar parallax from the angle $a''=6''{\cdot}50$, a value which was found by Le Verrier from a discussion of observations of the Sun between 1804 and 1850, made at Greenwich, Paris, and Königsberg.

Some doubts have been expressed about the use by Le Verrier of the argument $A=V-\odot$ for the determination of the angle $6''{\cdot}50$. But Le Verrier states most clearly, on pages 47 and 48 of his *Solar Tables*, that r', v', s', r, v are the coordinates at the time; and he actually works out an example of the use of his formulæ for 1847 April 8, mean noon, in which the true longitudes and not the mean longitudes are carefully used.

He employs for the determination of the solar parallax the factor which would be the correct factor if his coefficient was determined strictly in accordance with his formula; and after

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he has obtained the coefficient 6·50 he tabulates it for his *Solar Tables*, from an expression which is sensibly the same as that which I have found. (See pages 108, 109).

$$\begin{aligned} dv'' &= 6''{\cdot}50 \sin \mathrm{A} - 6''{\cdot}50[0{\cdot}0546 \,.\, \cos l + 0{\cdot}015 \cos 2l \\ &\quad + 0.0076 \cos 2\mathrm{D} + 0{\cdot}0096 \cos (2\mathrm{D} - l) + \&c.] \sin \mathrm{A} \\ &\quad + 6''{\cdot}50 \sin \mathrm{A} \times 0{\cdot}0169 \cos l' + 6''{\cdot}50 \,.\, \sin \mathrm{A} \,.\, \cos 2\mathrm{F}^* \end{aligned}$$

and in tabulating the results for his *Solar Tables* Le Verrier most carefully allows for the difference between A = V − ⊙ and D, or for the difference between the true and mean longitudes of the Moon and Sun.

It is incredible to me that Le Verrier should have taken all the necessary precautions in working out an example of the use of the formulæ and in tabulating the results; should have selected a factor which is the proper one when the true longitudes are used and for cos s' the mean value 0·99867 is substituted; and yet should have neglected the precautions necessary to obtain an accurate value of the angle from a discussion of such an extended series of observations as that employed for the direct object of a determination of the value of the solar parallax.

I have also developed the expression for sin A, and have thus been led to the expression

$$dv = \frac{\mu\pi}{\sin 1'' \,.\, \mathrm{C}} \, 1{\cdot}004451 \left[\begin{aligned} &\sin \mathrm{D} + 0{\cdot}0024 \sin 3\mathrm{D} \\ &- 0{\cdot}0662 \sin (\mathrm{D} - l) + 0{\cdot}0278 \sin (\mathrm{D} + l) \\ &+ 0{\cdot}0060 \sin (3\mathrm{D} - l) \\ &+ 0{\cdot}0272 \sin (\mathrm{D} - l') \\ &- 0{\cdot}0101 \sin (\mathrm{D} + l') \\ &+ 0{\cdot}0021 \sin (\mathrm{D} - 2\mathrm{F}). \end{aligned} \right]$$

In this case, if β is the angle found from a discussion of observations as the coefficient of sin D, we shall have

$$\pi = \frac{\beta}{\mu} \,.\, \frac{\mathrm{C} \sin 1''}{1{\cdot}004451};$$

or, adopting as before, C = 3422·7, we have

$$\pi = \frac{\beta}{\mu} \times 0{\cdot}01652,$$

a result which agrees very closely indeed with that which Professor Newcomb has obtained when β is supposed to be the coefficient of the term sin D.

My result, however, differs from that obtained by Professor Newcomb with respect to the term + 0·0021 sin (D − 2F), in the place of which Professor Newcomb has a term − 0·0020 sin 2F. I will, however, re-examine my work on this point; but I believe it to be correct.

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* This term is not included in Le Verrier's expression.

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It need hardly be pointed out that the angles α and β are not identical; and that the use of the factor 0·01652 with the angle α or the factor 0·01660 with the angle β will lead to sensibly erroneous values in the solar parallax.

The coefficients α or β will not however be obtained with accuracy from observations extending over periods of only two or three lunations, unless the periodical inequalities are accurately computed for the times of observation. The use of tabular places of the Sun, computed for intervals of a few days and then interpolated for the time of observation, will not in such cases be sufficient to secure accuracy.

The Secular Perturbations of the Earth arising from the Action of Venus. By R. T. A. Innes.

Although the eccentricities of the Earth and *Venus* are very small, the perturbing function is but slowly convergent on account of the high ratio of their mean distances.

Thus if the secular portion of the perturbing function be represented by

$$\mathrm{M}\, e^{\mathrm{h}}\, e^{\mathrm{h}}\, n^{\mathrm{f}} \cos \mathrm{A},$$

as in Le Verrier (*Annales*, tome ii. p. 89), the largest coefficient M is in the case of

$$\left.\begin{array}{ll} \text{Second power} = & 3{\cdot}2 \\ \text{Fourth power} = & 96{\cdot}9 \\ \text{Sixth power} = & 4710{\cdot}0 \end{array}\right\}\ \text{Le Verrier}$$

I therefore undertook the computation of the secular terms by Gauss's method, as laid down by Professor Hill. The elements used are found in Hill's "Jupiter and Saturn." The mass of *Venus* is assumed $\frac{1}{408134}$, and I get the following results. For comparison I add the figures found by Le Verrier and Hill by algebraical expansion, reducing all results to the above mass.

		Le V.	Hill.
	″	″	″
$\frac{de}{dt} =$	+ 0·013156	+ 0·013440	
$e\frac{d\varpi}{dt} =$	+ 0·057915	+ 0·057963	
$\frac{dp}{dt} =$	+ 0·074459	+ 0·074504	+ 0·074433
$\frac{dq}{dt} =$	− 0·284623	− 0·284538	− 0·284528
$\frac{dl}{dt} =$	+ 11·232490	+ 11·229806	